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HOMOMORPHISM AND ANTI-HOMOMORPHISM OF REVERSE DERIVATIONS ON PRIME RINGS

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Abstract— In this paper we show that if a reverse derivation d acts as a homomorphism or an anti-homomorphism on a non-zero right ideal U of a prime ring R, then $d = 0$.

Index Terms— Derivation, Reverse derivation, Prime ring, Center.

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1 Introduction

Macdonald [2] established some group-theoretic results in terms of inner derivations. Bell and Kappe [3] studied the analogous results for rings in which derivations satisfy certain algebraic conditions. I. N. Herstein [1] has introduced the concept of reverse derivations of prime rings and proved that a non-zero reverse derivation * of a prime ring A is a commutative integral domain and * is an ordinary derivation of A. Bresar, Vukman [4] and Samman, Alyamani [5] have studied some properties of prime (or) semi prime rings with reverse derivations. In this paper we show that if a reverse derivation d acts as a homomorphism or an anti-homomorphism on a non-zero right ideal U of a prime ring R, then $d = 0$.

2 PRELIMINARIES

We know that an additive map d from a ring R to R is called a derivation on R if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. A ring R is called prime if $xay = 0$ implies $x = 0$ or $y = 0$, for all x, a, y in R. An additive mapping d from a ring R into itself satisfying $d(xy) = d(y)x + yd(x)$, for all $x, y \in R$, is called a reverse derivation on R. Throughout this paper R will denote a prime ring and Z its center.

3 MAIN RESULTS

Theorem 1:
Let R be a prime ring and U a non-zero right ideal of R. If d is a reverse derivation of R which acts as a homomorphism (or) an anti-homomorphism on U, then $d = 0$ on R.

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Theorem 2: Let R be a prime ring and U a non-zero right ideal of R. Suppose $d : R \to R$ is a reverse derivation of R,

(i) If d acts as a homomorphism on U, then $d = 0$ on R.

(ii) If d acts as an anti-homomorphism on U, then $d = 0$ on R.

Proof: If d acts as a homomorphism on U, then we have,

$d(y)d(x) = d(yx) = d(x)y + xd(y)$, for all $x, y \in U$. ....(1)

we replace $y = yx$ in equ.(1), then, $\Rightarrow d(yx)d(x) = d(x)yx + xd(yx)$, for all $x, y \in U$. .... (2)

We multiply equ.(1) with $d(x)$ on the right hand side and using d is a homomorphism on U, then we get, $d(yx)d(x) = d(x)yd(x) + xd(y)d(x)$,

$d(yx)d(x) = d(x)yd(x) + xd(yx)$ (3)

By combining equ.'s (2) and (3), we get, $\Rightarrow d(x)yx = d(x)yd(x)$ (4)

$\Rightarrow x = d(x)$

$\Rightarrow (d(x) - x) = 0$

$\Rightarrow (d(x) - x)d(x) = 0$

$\Rightarrow d(x^2) = xd(x)$

Since d is a reverse derivation, we have, $d(x)x = 0$.

By linearizing x by $x + y$, then,

$\Rightarrow d(x)y + d(y)x = 0$ for all $x, y \in U$. (5)

We replace y by yx in equ.(5), then,

$\Rightarrow d(x)yx = 0$, for all $x, y \in U$. (6)

By substituting x by sx in equ.(6), then we get,

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$\Rightarrow d(x)ysx = 0,$ for all $x, y \in U$. and $s \in R$

Thus for each $x \in U$, the primeness of R forces that either $d(x)y = 0$ (or) $x = 0$.

But $x = 0$ also implies that $d(x)y = 0$, for all $x, y \in U$. (7)

If we replace y by ry in equ.(7), then we get,

$\Rightarrow d(x)ry = 0,$ for all $x, y \in U$ and $r \in R$

$\Rightarrow d(x)Ry = 0$

$d(x) = 0,$ for all $y \in U$. (8)

We replace x by sx in equ.(8), then we get,

$\Rightarrow d(sx) = 0$

$\Rightarrow d(x)s + xd(s) = 0$

$\Rightarrow xd(s) = 0,$ for all $x \in U$ and $s \in R$ ……………. (9)

Again replacing x by xr in equ.(9), then we get,

$\Rightarrow xrd(s) = 0,$ for all $x \in U$ and $r, s \in R.$

Hence $xRd(s) = \{0\},$

Since R is prime and U a non-zero right ideal of R, then $d = 0$ on R.

(i) If d acts as an anti-homomorphism on U.

By our hypothesis, we have,

$\Rightarrow d(yx) = d(x)d(y) = d(x)y + xd(y),$ for all $x, y \in U$ …………… (10)

By substituting yx for x in equ.(10), then, $\Rightarrow d(yx)d(y) = d(yx)y + yxd(y),$ for all $x, y \in U$ (11)

From equ.(10) implies that

$\Rightarrow xd(y)d(y) = yxd(y)$ (12)

We replace x by rx in equ.(12), then

$\Rightarrow rxd(y)d(y) = yrxd(y),$ for all $x, y \in U$ and $r \in R$ (13)

We multiply equ.(12) with r from the left, then we get,

$\Rightarrow rxd(y)d(y) = ryxd(y)$ (14)

By combining equ.'s(13) and (14), we get,

$\Rightarrow yrxd(y) = ryxd(y)$

$\Rightarrow [ry - yr]xd(y) = 0$

$\Rightarrow [r, y]xd(y) = 0,$ for all $x, y \in U$ and $r \in R$ (15)

We replace x by sx in equ.(15), then

$\Rightarrow [r, y]sxd(y) = 0,$ for all $x, y \in U$ and $s, r \in R$.Hence $[r, y]Rxd(y) = \{0\},$ for all $x, y \in U$ and $r \in R$.Thus, for each $y \in U$, the primeness of R forces that either $[r, y] = 0$ (or) $xd(y) = 0$.Let $A = \{y \in U / xd(y) = 0,\ for\ all\ x \in U\}$ and $B = \{y \in U / [r, y] = 0,\ for\ all\ r \in R\}$. Then clearly A and B are additive subgroups of U, whose union is U. By Brauer's trick, we have $xd(y) = 0,$ for all $x, y \in U$ (or) $[r, y] = 0,$ for all $y \in U$ and $r \in R$. If $[r, y] = 0,$ we replace y by sy, then $[r, sy] = 0$ which implies $[r, s]y = 0,$ for all $y \in U$ and $r, s \in R$. Therefore $[r, s]Ry = \{0\}$.The primeness of R forces either $y = 0$ (or) $[r, s] = 0,$ But $U \neq \{0\},$ then we have $[r, s] = 0,$ for all $r, s \in R$, that is R is commutative. So, $d(xy) = d(y)x + yd(x),$ for all $x, y \in U$ which implies that d is a reverse derivation which acts as an anti-homomorphism on U. Hence by Theorem: 1, we have $d = 0$ on R. Thus we have remaining possibility that

$xd(y) = 0,$ for all $x, y \in U$ (16)

If we replace x by xr in equ.(16), then we get,

$\Rightarrow xrd(y) = 0,$ for all $x, y \in U$ and $r \in R$.Hence $xRd(y) = 0$, which implies that,

$\Rightarrow d(y) = 0$, for all $y \in U$ (17)

By substituting sy for y in equ.(17), then we obtain,

$\Rightarrow d(sy) = 0$

$\Rightarrow d(y)s + yd(s) = 0$

$\Rightarrow yd(s) = 0$, for all $y \in U$ and $s \in R$ (18)

We replace y by yr in equ.(18), then

$\Rightarrow yrd(s) = 0$, for all $y \in U$ and $r, s \in R$.

Hence $yRd(s) = \{0\}$.

Since R is prime and U a non-zero right ideal of R, then $d = 0$ on R

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